LOGARITHMIC CALCULUS

GOSSA AHMED
gossaahmed@yahoo.fr

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1-preface:

we have two method to understand something the first method that is determined us the points of deference of this thing and an others thing that we understand well, the second method it is of determined the points of correspondence between this thing and another one that understand us well about this article to understand logarithmic calculus we us adapted the second method that is has say makes a comparison of it with deferential calculus and proportional calculus

1-a-differential calculus :
The main objective of deferential calculus is to calculate the deference of all quantities varies we called mathematical functions is that by creating relationships between the infinitely small dy1. dy2... ... ... dyn variations of its functions with infinitely small dx independent of the value x following functions can be seen that in the infinitely small that is to say dy dx tends to 0 is the only right relationship between the deference dy and dx is a relationship of multiplication:

$$dy = y'(x)dx$$

(y') called deferential derivative connecting the deference dy at deference dx We can calculate the quantity of dy with a simple algebraic operation if the (y') remains constant with dx. We can not calculate the quantity of dy when (y') variable with dx at simple algebraic operation Leibniz and newton had established a new calculus By which we can calculate the quantity dy when (y') variable with dx is called deferential calculus the deferential calculus which has made a revolution in the science of mathematics and its applications in the field of physics and chemistry and other In addition, this knowledge led to the emergence of new concepts such as physical sciences and chemical and other the concept of continuity and the infinitely small and derivatives and integration Based on the philosophical principle that says that each quantity is a collection of Quantities infinitesimals deferential Calculus said that the deference and D equal the sum of deference infinitesimal di which can be expressed dy the following mathematical expression: ... .

$$D = d_1 + d_2 + d_3 + d_n$$

$$d_1 = y'(x_1)dx_1$$

$$d2 = y'(x_2)dx_2$$
$$dn = y'(x_n)dx_n$$

It is through this calculus we managed to calculate areas and volumes and other quantities which can not be calculated by simple algebraic operations. It is through this calculus we finally managed to establish that equations called differential equations by the solutions of its equations we found the functions of all physical quantities such as heat and pressure and forces ... ... After this brief summary of differential calculus, we have the right to ask is this-that the deferential calculus and the only one of the mathematical calculation

1-b-proportional calculus :
The main purpose of proportional calculus is it to calculate the proportions that we denote by R of all quantity that varied called mathematical functions is that by creating relationship between proportions that we denote by ry $ry_1.ry_2..........ry_n$ tends to 1 with proportions $rx_1.rx_2.......rx_n$ that tends to 1 as it notes that the value of rx independent of the value x after its functions we see that in the field of proportions tends to the 1 that is to say, tends to 1 ry rx tends to 1 is the only right relationship between these proportions ry and rx is an exponential relationship that is to say.

$$ry = (rx)^{y^*(x)}$$

$y^*(x)$is called the proportional derivative connects the proportions (rx) and (ry) with the proportion that the derivative called the proportional derivative can be constant or varied. So the proportion (R) equals the multiplication of a (ry):

$$R = \prod_{i=1}^{i=\infty} ry_i$$

Which can be calculate with him surfaces and forces

Is that-with this calculation can only be expressed with it the chemical physics equations statistics... ... . Finally,do this deference calculus and proportional calculus is the only languages of physical science and chemical ... . these calculations is not the only one with which we can express the physical and chemical laws there are several mathematical calculations we can express the physical and chemical laws and other science among these calculations it is of the logarithmic calculus

In what follows we are interested only in the functions of one variable x as well all numbers are real numbers that is to say all x in R. and the applications proportional calculus of the functions of several variables and complex numbers we apply them in another document. Also demonstrations of formulas and calculation steps was the fact when you asked me what I want in documents by not only essential to the calculation with concepts such as recalculation logarithmic derivative. Theorem of the mean logarithmic. logarithmic integral . development limited by the series of logarithmic. which gives the approximation faster than Taylor series differential. Also as in the calculus has created equations called the logarithmic equations so we will study it in this text and we will make a few resolutions Examples

2-principle of logarithmic calculus :
The logarithmic calculus is based on the principle that says that every mathematical logarithm is $(log_a b)$ we denote L equals the multiplication of infinite logarithm (l_i) do $(l_i = log_{x_i} x)$ x tend to x_i which is expressed mathematical by the following equation:

$$L = l_1.l_2.l_3.....l_n$$

$$l_1 = log_{x_1} x$$

$$x \to x_1$$

and

$$l_2 = log_{x_2} x$$

$$x \to x_2$$

and

$$l_n = log_{x_n} x$$

$$x \to x_n$$

3-notation :
in all that follows note himself us derivative deferential the number it that joined the deference of the variation $(y2 - y1)$ function with the variation $(x2 - x1)$ when x2 stretches toward x1 this number is varied or constant one

calls as derivative proportional the number that joined the report of the function $f(x2)/f(x1)$ with the variation $(x2/x1)$ when x2 stretches toward x1 we also note derivative logarithmic numbers it that joined logarithm $f(x2)$ by report bases it $f(x1)$ on note par($log_{f(x2)}f(x1)$) with the variation logarithm $x2$ by report bases it $x1$ we note by ($log_{x2}x1$)when x2 stretches toward x1

4-logarithmic derivative :
as in deferential calculus one has find a relation between dy with dx when dy dx will be infinitely small is this relation himself call deferential derivative. also as in proportional calculus one has find a relation between ry and rx is this relation himself call proportional derivative. therefore in logarithmic calculus one makes the same thing that is has say to find a relation that joins ly and lx or $ly = log_{y_i}y\ y \to y_i$ and $lx = log_{x_i}x\ x \to x_i$

] let f(x) interval containing x we say that f (x) has logarithmic derivative to x if the quantity

$$\frac{ln(lf(x))}{ln(lx)}$$

admits a finite limit called logarithmic derivative at number x we denote this limit by f(x) which given following equation:

$$lf(x) = (l(x))^{f^{\circledast}(x)}$$

$f^{\circledast}(x)$:logarithmic derivative

we have

$$lf(x) = \frac{lnf(x)}{lnf(x_1)}$$

as x1 tend at x and

$$lx = \frac{lnx}{lnx_1}$$

as x1 tend at x

5-relation between logarithmic derivative and proportional derivative and deferential derivative :
of after calculate them one has find the following relations

$$y' = \frac{dy}{dx}$$

$$y^* = y'\frac{x}{y}$$

$$y^{\circledast} = y' \frac{x}{y} \frac{lnx}{lny}$$

$$y^{\circledast} = y^{*} \frac{lnx}{lny}$$

6-logarithmic derivative of high order :
As in the deferential calculus and proportional calculus are derivative from higher-order functions

$$f', f'', f''', ...f^{n}$$

$$f^{*}.f^{**}.f^{***}...........f^{*n}$$

in the logarithmic calculus exist also the logarithmic derivative of high order :

$$f^{\circledast}.f^{\circledast\circledast}.f^{\circledast\circledast\circledast}...........f^{\circledast}n$$

7- logarithmic derivative of some functions:

$$f(x) = e^{x} \Rrightarrow f^{\circledast}(x) = lnx \Rrightarrow f^{\circledast\circledast}(x) = \frac{1}{ln(lnx)}$$

$$f(x) = x^{2} \Rrightarrow f^{\circledast}(x) = 1 \Rrightarrow f^{\circledast\circledast}(x) = 0$$

$$f(x) = x^{n} \Rrightarrow f^{\circledast}(x) = 1 \Rrightarrow f^{\circledast\circledast}(x) = 0$$

$$f(x) = x \Rrightarrow f^{\circledast}(x) = 1$$

$$f(x) = x^{lnx} \Rrightarrow f^{\circledast}(x) = 2$$

$$f(x) = x^{(lnx)^{2}} \Rrightarrow f^{\circledast}(x) = 3$$

$$f(x) = x^{(lnx)^{3}} \Rrightarrow f^{\circledast}(x) = 4$$

$$f(x) = x^{(lnx)^{4}} \Rrightarrow f^{\circledast}(x) = 5$$

$$f(x) = x^{(lnx)^{n}} \Rrightarrow f^{\circledast}(x) = n + 1$$

6-operation on the logarithmic derivative :
Let f and g be two real functions of one variable and let $f(x)$ and $g(x)$ is different:

$$(f(x)^{g(x)})^{\circledast} = g^{\circledast}(x).lng(x) + f^{\circledast}(x)$$

$$(f(x).g(x))^{\circledast} = \frac{f^{\circledast}(x)}{1+\frac{lng(x)}{lnf(x)}} + \frac{g^{\circledast}(x)}{1+\frac{lnf(x)}{lng(x)}}$$

fundamental relationship between the proportional derivative and logarithmic derivative changes in the subtraction. Following are true:
After some simple calculation we find the following relation which links the logarithmic derivative $f^{\circledast}(x)$ and that $f(x)$ and that proportional derivative $f^{\star}(x)$ and x this independent value

$$f(x)^{f^{\circledast}(x)} = x^{f^{\star}(x)}$$

relationship is very important Especially in solving differential equations . When we convert the equations proportional And also to calculate the derivatives according to the proportional derivative differential

example:

$f(x) = x^5$ therefore $f^{\circledast}(x)) = 1$ and $f^{*}(x) = 5$ therefore $(x^5)^1 = x^5$

8-Examples of logarithmic derivatives :
Calculate the derivatives of the following functions proportional:

$$f(x) = sinx$$

the logarithmic derivative is:

$$f^{\circledast}(x) = x.\frac{cosx}{sinx}.\frac{lnx}{ln(sinx)}$$

$$f(x) = lnx$$

the logarithmic derivative is:

$$f^{\circledast}(x) = \frac{1}{ln(lnx)}$$

9-Theorem of the logarithmic mean:
Let f be a function defined on a segment [a, b] R
* f is continuous on [a, b]
* f is differentiable proportional] a, b [
then there exists at least one number c in] a, b [as:

$$log_{f(a)}f(b) = (log_a b)^{f^{\circledast}(c)}$$

since $log_{f(a)} f(b) = \frac{lnf(b)}{lnf(a)}$ and $log_a b = \frac{lnb}{lna}$therefore:

$$\frac{lnf(b)}{lnf(a)} = (\frac{lnb}{lna})^{f^{\circledast}(c)}$$

Example:

$$f(x) = e^x$$

. f (x) is continuous in the interval [10.8] was based on the theorem of the mean proportional :

$$(lnf(10))/ln(f(8)) = (ln10/ln8)^{f^{\circledast}(c)}$$

was

$$f^{\circledast}(x) = (e^x)^{\circledast} = lnx$$

$$f^{\circledast}(c) = ln(lnf(10)/lnf(8))/ln(ln10/ln8) \Rrightarrow c = 8,927 \in [10,8]$$

10-logarithmic series :
as in the deferential calculus we can makes an approximation of a tell function by a series of sum called series of Taylor.also as in the proportional calculus we can make an approximation of a function by a series of multiplication that one called series of proportion.

Taylor series(sum series):

$$f(x) - f(a) = f(a) + \frac{f^{'}(a)(x-a)}{1!} + \frac{f^{''}(a)(x-a)^2}{2!} + ...\frac{f^n(a)(x-a)^n}{n!}$$

(multiplicative series):

$$\frac{f(x)}{f(a)} = (\frac{x}{a})^{f^*(a)}.(\frac{x}{a})^{\frac{f^*(a).f^{**}(a).(ln(\frac{x}{a}))}{1!}}.(\frac{x}{a})^{\frac{f^*(a).f^{**}(a).(f^*(a).f^{**}(a))^*.(ln(\frac{x}{a}))^2}{2!}}.............$$

Note that the Taylor series is built by f deference of:

$$(f(x) - f(a))$$

equals the sum of deference independent value:

$$(x - a)$$

multiplied by its successive deferential derivatives at point a
Note that the proportional series is built by f proportion of:

$$(\frac{f(x)}{f(a})$$

equals the multiplication of proportion independent value:

$$(\frac{x}{a})$$

exposant by its successive proportional derivatives at point a
But in logarithmic calculus $f(x) - f(a)$ and $(\frac{f(x)}{f(a})$ is converted to the logarithm there:

$$log_{f(a)}f(x) = \frac{lnf(x)}{lnf(a)}$$

and the deference of the value(x-a) and the proportion (x/a) is converted to independent logarithm that is to say:

$$log_a x = \frac{lnx}{lna}$$

the derivative successive multiplication of differential derivative to convert to the exponent proportional derivative from successive we find the following relation So after simplification we can write the(exponential series):

$$log_{f(a)}f(x) = \frac{lnf(x)}{lnf(a)} = 1 + \frac{f^{\circledast}(a).ln(\frac{lnx}{lna})}{1!} + \frac{f^{\circledast}(a)((f^{\circledast}(a)lnf(a))^{\circledast}.ln(f^{\circledast}(a)lnf(a)).(ln(\frac{lnx}{lna}))^2}{2!} + .$$

So after simplification we can write it like this:

$$f(x) = (f(a))^{1+\frac{f^{\circledast}(a)ln(\frac{lnx}{lna})}{1!}+\frac{f^{\circledast}(a)((f^{\circledast}(a)f(a))^{\circledast}(ln(\frac{lnx}{lna}))^2}{2!}+\dots\dots\dots}$$

As done in the Maclaurin series of Taylor replaced (a) by 0 and in the Proportional series is replaced by 1 include in the logarithmic series is replaced by e include:

$$f(x) = (f(e))^{1+\frac{f^{\circledast}(e).ln(lnx)}{1!}+\frac{f^{\circledast}(e)((f^{\circledast}(e).lnf(e))^{\circledast}ln((f^{\circledast}(e).lnf(e))(ln(lnx))^2}{2!}+\dots\dots\dots}$$

therefore

$$f(x) = (f(e))^{1+\frac{f^{\circledast}(e).ln(lnx)}{1!}+\frac{f^{\circledast}(e)((f^{\circledast}(e).lnf(e))^{\circledast}.ln(f^{\circledast}(e).lnf(e)).(ln(lnx))^2}{2!}+\dots\dots\dots}$$

9- example of Development the function by the logarithmic series:

$$f(x) = \sqrt{x}$$

About the calculations are:

$$\sqrt{x} = (\sqrt{e})^{1+\frac{ln(ln(x))}{1!}+\frac{(ln(ln(x)))^2}{2!}+\frac{(ln(ln(x)))^3}{3!}+\frac{(ln(ln(x)))^4}{4!}+\frac{(ln(ln(x)))^5}{5!}..........+\frac{(ln(ln(x)))^n}{n!}}$$

Witt made a comparison with the Taylor series and proportional series and we take the seven terms first e the series

$$\sqrt{10000}$$

is calculated by the Taylor series:

$$\sqrt{(x+1)} = 1 + \frac{x}{2} - \frac{1.x^2}{8} + \frac{1.x^3}{16} - \frac{5.x^4}{128} + ...$$

To calculate

$$\sqrt{1000}$$

we set x = 999

$$\sqrt{1000} = 1+199,8-12475,125+62312687,43-7781296843,75 = -7719108705,64$$

is calculated by the proportional series:

$$\sqrt{(x)} = 1 + \frac{lnx}{2 \times 1!} + \frac{(lnx)^2}{4 \times 2!} + \frac{(lnx)^3}{8 \times 3!} + \frac{(lnx)^4}{16 \times 4!} + ... + \frac{(lnx)^n}{2^n \times n!}.$$

we set x = 1000

$$1 + 3,453 + 5,9646 + 6,867 + 5,929 = 23,2136$$

We see that

$$\sqrt{1000} = 23,2136$$

is calculated by the logarithmic series:

$$\sqrt{x} = (\sqrt{e})^{1+\frac{ln(ln(x))}{1!}+\frac{(ln(ln(x)))^2}{2!}+\frac{(ln(ln(x)))^3}{3!}+\frac{(ln(ln(x)))^4}{4!}+\frac{(ln(ln(x)))^5}{5!}..........+\frac{(ln(ln(x)))^n}{n!}}$$

we set x = 1000

$$\sqrt{1000} = (\sqrt{e})^{1+1,932+1,867+1,203+0,581+0,224+0,0723}$$

$$\sqrt{1000} = (\sqrt{e})^{6,8793}$$

$$\sqrt{1000} = (e)^{3,43965} = 31,176$$

According to the Taylor series we found:

$$\sqrt{1000} = -7719108705,64$$

According to the proportional series was found:

$$\sqrt{1000} = 23,2136$$

According to the logarithmic series was found:

$$\sqrt{1000} = 31,176$$

According to the Taylor series of the error calculation is

$$31.622 - (-7719108705,6) = 7719140327,6$$

According to the proportional error range

$$31.622 - (23,2136) = 8.409$$

According to the logarithmic series error range

$$31.622 - (31,176) = 0.44$$

Look at the great deference therefore the use of logarithmic series

10-logarithmic Primitive :

As in the deferential calculus and proportional calculus F(x) is primitive logarithmic of f(x) is that we derive F(x) in logarithm we find f (x) which is written mathematically by the following equation:

$$(F(x))^{\circledast} = f(x)$$

Examples:

$$f(x) = 1$$

the proportional primitive are:

$$F(x) = (x)^k$$

with k any real

$$f(x) = 2$$

the proportional primitive are:

$$F(x) = ((x)^{lnx})^k$$

with k any real

$$f(x) = lnx$$

- proportional primitive are:

$$F(x) = (e^x)^k$$

with k any real

$$f(x) = \frac{x}{x+1}\frac{lnx}{ln(x+1)}$$

the proportional primitive are:

$$F(x) = (x+1)^k$$

with k any real

$$f(x) = \frac{x}{tgx}\frac{lnx}{ln(sinx)}$$

proportional primitive are:

$$F(x) = (sinx)^k$$

with k any real

11-logarithmic integral :
To better understand the logarithmic integral we will compare it with the differential integral and proportional integral which we understand well. In the deferential calculus to calculate the integral of f (x) from c to d we calculate the differential primitive:

$$F(x)$$

then find the following deference:

$$(F(d) - F(c))$$

that it should be noted by the following mathematical relationship :

$$F(d) - F(c) = \int_c^d f(x)dx$$

is an infinite sum of deference that the worm tends (0) In the proportional calculus to calculate the integral of f (x) from c to d we calculate the proportional primitive:

$$F(x)$$

then find the following proportion:

$$\frac{F(d)}{F(c)}$$

that it should be noted by the following mathematical relationship :

$$\frac{F(d)}{F(c)} = \chi_c^d (rx)^{f(x)}$$

is an infinite multiplication of proportion that the worm tends to 1

In the logarithmic calculus to calculate the logarithmic integral of f (x) from c to d we calculate the logarithmic primitive F(x) then made calculate the following logarithm:

$$log_{F(d)} F(c)$$

it should be noted that the mathematical relationship follows:

$$log_{F(c)} F(d) = E_c^d (lx)^{f(x)}$$

designated the multiplication of infinite logarithm which tends to 1

We see that the deference in deferential calculus :

$$F(d) - F(c)$$

changes in proportion in proportional calculus:

$$F(d)/F(c)$$

and change in logarithm in logarithmic calculus:

$$log_{F(d)} F(c)$$

We see that the deference in deferential calculus:

$$dx$$

is transformed in proportion in proportional calculus:

$$rx$$

and is transformed in logarithm in logarithmic calculus:

$$log_d c$$

We change the sign of the sum in deferential calculus:

$$\int$$

by the new multiplication sign in proportional calculus:

$$\chi$$

and by the new exponent sign in logarithmic calculus:

$$E$$

has to distinguish the differential calculus and proportional calculus

example of logarithmic integral:
Calculates the logarithmic integral at the function:

$$f(x) = 2$$

2 to 4 we note that mathematically by the following equation:

$$E_2^4 (lx)^2$$

This integral equals:

$$log_{F(2)} F(4)$$

or F (x) is logarithmic primitive of f (x) We calculated the logarithmic primitive was equal:

$$F(x) = (x^{lnx})^k$$

$$log_{F(2)} F(4) = \frac{lnF(4)}{lnF(2)} = \frac{(ln4)^2}{(ln2)^2} = 4$$

So the logarithmic integral at 2 to 4 equal: 4

proportional integral laws:

$$E_a^a(lx)^{f(x)} = 1$$
$$(\forall f(x), a, \in \mathbb{R})$$
$$E_a^b(lx)^{f(x)}.E_b^a(lx)^{f(x)} = 1$$
$$\forall f(x), a, b, \in \mathbb{R}$$

logarithmic equations:
equations are logarithmic to the relationships that bind a function f (x) with logarithmic derivatives:

$$F(x.f.f^{\circledast}.f^{\circledast\circledast}...............f^{\circledast n})$$

by mathematical equations:.....................

$$F(f.f^{\circledast}.f^{\circledast\circledast}.f^{\circledast\circledast\circledast}..................f^{\circledast n}) = R(x)$$

. Examples of proportional equations:

$$y^{\circledast}x + y = 0$$

y are unknown

$$y^{\circledast\circledast} + y^{\circledast}x + y = tg(x)$$

y are unknown

5-relation between logarithmic integral and proportional integral and deferential integral :
after the calculations we find the following important relationships:

$$\chi_b^a(rx)^{f(x)} = e^{\int_b^a \frac{f(x)}{x}dx}$$
$$E_b^a(lx)^{f(x)} = e^{\chi_b^a(rx)^{\frac{f(x)}{lnx}}}$$